CHAPTER 3

Natural Gas Production

3.1 Introduction

Once the well is drilled and completed successfully, it is ready to produce fluids (assuming the oil and gas-in-place are there and it is economical to operate the well). The produced hydrocarbons in the gaseous phase are from two main sources of natural gas (as discussed in Chapter 1).

First, gas is found in association with oil. Almost all oil reservoirs, even those that are insitu above their bubble point pressure, will shed some natural gas, which is produced at the surface with oil and then separated in appropriate surface facilities. The relative proportions of produced gas and oil depend on the physical and thermodynamic properties of the specific crude oil system, the operating pressure downhole, and the pressure and temperature of the surface separators.

The second type of gas is produced from reservoirs that contain primarily gas (dry gas or gas condensate). Usually such reservoirs are considerably deeper and hotter than oil reservoirs. We will deal with the production characteristics of these reservoirs in this chapter.

There are other unconventional sources of natural gas, one of which is coalbed methane desorbed from coal formations, and already in commercial use. The process is described in Chapter 11 of Economides and Martin (2007). In the far future, production from massive deposits of natural gas hydrates is likely, but such eventuality is outside the scope of this book.

In this chapter, gas well performance and deliverability at different flow conditions—steady state, pseudosteady state, and transient flow—under Darcy and non-Darcy flow with and without hydraulic fractures will be discussed.
3.2 Darcy and non-Darcy Flow in Porous Media

To perform natural gas well deliverability calculations, it is essential to understand the fundamentals of gas flow in porous media. Fluid flow is affected by the competing inertial and viscous effects, combined by the well-known Reynolds number, whose value delineates laminar from turbulent flow. In porous media, the limiting Reynolds number is equal to 1 based on the average grain diameter (Wang and Economides, 2004).

Because permeability and grain diameter are well connected (Yao and Holditch, 1993), for small permeability values (e.g., less than 0.1 md) the production rate is generally small; flow is laminar near the crucial sandface and it is controlled by Darcy’s law:

\[-\frac{dp}{dx} = \frac{\mu_s}{k_s} v_s,\]  

(3.1)

where \(x\) represents the distance, \(p\) the pressure, \(v_s\) the gas velocity, \(\mu_s\) the gas viscosity and \(k_s\) the effective permeability to gas. An amount of connate water is always present with the gas. Such water saturation is immobile and, therefore, \(k_s\) equals the effective permeability to gas and can be treated as the single-phase permeability. It is often denoted simply as \(k\).

Non-Darcy flow occurs in the near-wellbore region of high-capacity gas and condensate reservoirs: As the flow area is reduced substantially, the velocity increases, inertial effects become important, and the gas flow becomes non-Darcy. The relation between pressure gradient and velocity can be described by the Forchheimer (1914) equation

\[-\frac{dp}{dx} = \frac{\mu_s}{k_s} v_s + \rho_g \beta_g v_s^2,\]  

(3.2)

where \(\rho_g\) is the gas density. \(\beta_g\) is the effective non-Darcy coefficient to gas. It can be calculated by using published theoretical or empirical correlations. Table 3–1 is a summary of some of the correlations. These correlations are valid for single-phase gas flow (subscript “\(g\)” is dropped for simplicity).

It is worth noting that condensate liquid may flow if its saturation is above the critical condensate saturation (\(S_{cc}\)) (Wang and Mohanty, 1999a). Additional condensate drops out because the further reduced
### Table 3–1 Correlations for non-Darcy Coefficient

<table>
<thead>
<tr>
<th>Reference</th>
<th>Correlation</th>
<th>Unit for $\beta$</th>
<th>Unit for $k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooke (1973)</td>
<td>$\beta = \frac{b}{k^a}$</td>
<td>atm.s$^2$/g</td>
<td>darcy</td>
</tr>
<tr>
<td>Thauvin &amp; Mohanty (1998)</td>
<td>$\beta = \frac{3.1 \times 10^4 \tau^3}{k}$</td>
<td>1/cm</td>
<td>darcy</td>
</tr>
<tr>
<td>Geerstma (1974)</td>
<td>$\beta = \frac{0.005}{k^{0.5} \phi^{0.5}}$</td>
<td>1/cm</td>
<td>cm$^2$</td>
</tr>
<tr>
<td>Tek et al. (1962)</td>
<td>$\beta = \frac{5.5 \times 10^9}{k^{1.25} \phi^{0.75}}$</td>
<td>1/ft</td>
<td>md</td>
</tr>
<tr>
<td>Liu et al. (1995)</td>
<td>$\beta = \frac{8.91 \times 10^8 \tau}{k\phi}$</td>
<td>1/ft</td>
<td>md</td>
</tr>
<tr>
<td>Ergun (1952)</td>
<td>$\beta = \frac{a}{b^{0.5}(10^{-8} k)^{0.5} \phi^{1.5}}$</td>
<td>1/cm</td>
<td>darcy</td>
</tr>
<tr>
<td>Janicek &amp; Katz (1955)</td>
<td>$\beta = \frac{1.82 \times 10^8}{k^{1.25} \phi^{0.75}}$</td>
<td>1/cm</td>
<td>md</td>
</tr>
<tr>
<td>Pascal et al. (1980)</td>
<td>$\beta = \frac{4.8 \times 10^{12}}{k^{1.176}}$</td>
<td>1/m</td>
<td>md</td>
</tr>
<tr>
<td>Jones (1987)</td>
<td>$\beta = \frac{6.15 \times 10^{10}}{k^{1.55}}$</td>
<td>1/ft</td>
<td>md</td>
</tr>
<tr>
<td>Coles &amp; Hartman (1998)</td>
<td>$\beta = \frac{1.07 \times 10^{12} \times \phi^{0.449}}{k^{1.88}}$</td>
<td>1/ft</td>
<td>md</td>
</tr>
<tr>
<td>Coles &amp; Hartman (1998)</td>
<td>$\beta = \frac{2.49 \times 10^{11} \phi^{0.537}}{k^{1.79}}$</td>
<td>1/ft</td>
<td>md</td>
</tr>
<tr>
<td>Li et al. (2001)</td>
<td>$\beta = \frac{11500}{k\phi}$</td>
<td>1/cm</td>
<td>darcy</td>
</tr>
<tr>
<td>Wang et al. (1999)</td>
<td>$\beta = \frac{(10)^{-3.25} \tau^{1.943}}{k^{1.023}}$</td>
<td>1/cm</td>
<td>cm$^2$</td>
</tr>
</tbody>
</table>

$\tau$ is tortuosity
pressure will aggravate the situation. Therefore, two phenomena emerge: non-Darcy effects and a substantial reduction in the relative permeability to gas. Because of the radial nature of flow, the near-wellbore region is critical to the productivity of a well. This is true in all wells, but it becomes particularly serious in gas-condensate reservoirs.

Forchheimer’s equation describes high-velocity, single-phase flow in isotropic media. Many reservoirs are, however, anisotropic (Wang et al., 1999; Wang, 2000). Wang (2000) used a pore-level model and developed a correlation to calculate the non-Darcy coefficient in an anisotropic medium for single-phase flow (see Table 3–1). Cooper et al. (1998) studied the non-Darcy coefficient by performing experimental tests with carbonate and Berea sandstone cores. Their experimental data gave good agreement with the correlation described by Wang (2000).

A direct understanding of multiphase non-Darcy flow behavior in porous media that are anisotropic at the pore-scale is studied elsewhere (Wang, 2000; Wang and Mohanty, 1999b).

3.3 Gas Well Inflow under Darcy Flow

Well inflow means the fluid flow from the reservoir into the sandface, takes into account the reservoir characteristics, the well geometry (vertical, horizontal, complex architecture), the near-wellbore zone or other features such as hydraulic or natural fractures and the pressure drawdown. Different flow regimes that take into account boundary effects such as steady state, pseudosteady state and transient behavior are considered.

Natural gas well performance will be discussed in the following sections, based on its flow characteristics under different flow regimes.

3.3.1 Steady State and Pseudosteady State Flow

Steady state flow is defined as the behavior when the pressure (wellhead or bottomhole) and flow rates are constant. This behavior usually happens when there is pressure support, either naturally through an aquifer, or through water injection. The well performance under steady state flow can be derived from Darcy’s law.

Starting with a well in the center of a drainage, as shown in Figure 3–1, with \( r_w \) the wellbore radius, \( p_{wf} \) the flowing bottomhole pressure, \( p \) the pressure at any given distance \( r \), and with the net reservoir thickness \( h \), the cross-sectional flow area can be calculated as \( 2\pi rh \). In radial coordinates, Eq. (3.1) becomes
3.3 Gas Well Inflow under Darcy Flow

The flow rate $q$ is constant as the flow is under steady state. Eq. (3.3) can be integrated by separating the variables and setting at the outer boundary $r_e$, a constant pressure $p_e$:

$$p_e - p_{wf} = \frac{q \mu}{2 \pi k h} \ln \frac{r_e}{r_w}.$$  \hfill (3.4)

Van Everdingen and Hurst (1949) quantified the condition of the near-wellbore region with the introduction of the concept of the skin effect. This is analogous to the film coefficient in heat transfer. This skin effect results in an additional steady-state pressure drop, given by

$$\Delta p_s = \frac{q \mu}{2 \pi k h} s.$$  \hfill (3.5)

Thus, Eq. (3.4) can provide the total pressure difference including both the reservoir and the near-wellbore zone and becomes

$$p_e - p_{wf} = \frac{q \mu}{2 \pi k h} \left( \ln \frac{r_e}{r_w} + s \right).$$  \hfill (3.6)
In oilfield units, where \( p_e \) and \( p_{wf} \) are in psi, \( q \) is in stb/d, \( \mu \) is in cp, \( k \) is in md, \( h \) is in ft, \( s \) is dimensionless, and \( B \) is the formation volume factor to convert reservoir barrel (res bbl) into stock tank barrel (stb), Eq. (3.6) yields

\[
p_e - p_{wf} = \frac{141.2qB\mu}{kh} \left( \ln \frac{r_e}{r_w} + s \right).
\]  

(3.7)

Eq. (3.7) is valid for largely incompressible (i.e., oil) flow under steady state. For highly compressible gas, the formation volume factor, \( B_g \), varies greatly with pressure. Therefore an average expression can be obtained from Eq. (1.12),

\[
\frac{B_g}{B_o} = \frac{0.0283ZT}{(p_e + p_{wf} ) / 2}.
\]  

(3.8)

Introducing the gas rate in Mscf/d (thousand standard cubic feet per day), with relatively simple algebra, Eq. (3.7) yields

\[
p_e - p_{wf} = \frac{141.2(1,000 / 5.615)q(0.0283)ZT\bar{\mu}}{[(p_e + p_{wf}) / 2]kh} \left[ \ln \left( \frac{r_e}{r_w} \right) + s \right],
\]  

(3.9)

and finally

\[
p_e^2 - p_{wf}^2 = \frac{1,424q\bar{\mu}ZT}{kh} \left[ \ln \left( \frac{r_e}{r_w} \right) + s \right],
\]  

(3.10)

which, re-arranged, provides the steady-state approximation for natural gas flow, showing a pressure-squared difference dependency

\[
q = \frac{kh(p_e^2 - p_{wf}^2)}{1,424\bar{\mu}ZT[\ln \left( \frac{r_e}{r_w} \right) + s]},
\]  

(3.11)

where the properties \( \bar{\mu} \) and \( \bar{Z} \) are average properties between \( p_e \) and \( p_{wf} \) (henceforth the bars will be dropped for simplicity).

Eq. (3.11) is valid for gas flow under steady state (with a constant-pressure outer boundary). More commonly, wells eventually feel their
assigned boundary. Drainage areas can either be described by natural limits such as faults, and pinchouts (no-flow boundary), or can be artificially induced by the production of adjoining wells. This condition is often referred to as “pseudosteady state”. The pressure at the outer boundary is not constant but instead declines at a constant rate with time, that is, \( \frac{\partial p}{\partial t} = \text{const.} \). Therefore, a more useful expression for the pseudosteady-state equation would be one using the average reservoir pressure, \( \bar{p} \). It is defined as a volumetrically weighted pressure (Economides et al., 1994) and in practice can be obtained from periodic pressure buildup tests.

The production rate expression for a gas well can be written for pseudosteady state,

\[
q = \frac{kh(\bar{p}^2 - p_{wf}^2)}{1,424\mu ZT[\ln\left(\frac{0.472r_s}{r_w}\right) + s]} .
\] (3.12)

Eqs. (3.11 and 3.12) suggest a number of interesting conclusions: the flow rate is large if the pressure-squared difference is large, if the permeability and reservoir net thickness are large or the gas deviation factor, the viscosity of the flowing fluid, and the skin damage are small. It is clear that a positive skin means the well is damaged and this will cause additional pressure drop in the near wellbore region. A negative skin means the well is stimulated (through matrix acidizing and removing near-wellbore damage, or through hydraulic fracturing by bypassing the damage zone and changing flow paths).

In summary, Eq. (3.12) (or Eq. (3.11)) is an analytical approximation of gas well rate under pseudosteady (or steady) state and Darcy flow conditions in the reservoir. It is valid when gas flow rate is small. It can be presented in a common form

\[
q = C(\bar{p}^2 - p_{wf}^2) .
\] (3.13)

A log-log plot of \( q \) versus \( (\bar{p}^2 - p_{wf}^2) \) would yield a straight line with slope equal to one and intercept \( C \). For large flow rates, non-Darcy flow will be present in the reservoir. This will be addressed in a later section of this chapter.
Example 3–1 Rate versus pressure

Consider a gas reservoir whose pressure is 3,000 psi. Assess the impact of the flowing bottomhole pressure on flow rate. Assume a steady-state relationship and use $p_{wf} = 2,500$, $2,000$, $1,500$, $1,000$, and $500$ psi, respectively. Given,

- $p_e = 3,000$ psi
- $r_e = 660$ ft
- $r_w = 0.359$ ft
- $k = 0.1$ md
- $h = 50$ ft
- $T = 250$ °F
- $g = 0.7$
- $N_2 = 0$
- $CO_2 = 0$
- $H_2S = 0$
- $s = 0$

Solution

Eq. (3.10) after substitution of variables becomes

$$9 \times 10^6 - p_{wf}^2 = (1.52 \times 10^6)q\overline{\mu}Z \cdot$$

Gas viscosity and Z-factor at different flowing bottom pressures are calculated by using Lee et al. (1966) and Dranchuk et al. (1974) correlations (presented in Chapter 1), respectively. The average properties are the arithmetic average with properties at $p_e$ of 3,000 psi. Results are summarized in Table 3–2.

As an example calculation, for $p_{wf} = 1,000$ psi, the above equation yields

$$q = \frac{9 \times 10^6 - 1,000^2}{1.52 \times 10^6 \times 0.0176 \times 0.923} = 324 \text{ Mscf/d}.$$
Figure 3–2 is a graph of $p_{wf}$ versus $q$ for this example. It shows the flow rate increases when the $p_{wf}$ decreases as the driving force ($p_e^2 - p_{wf}^2$) increases. If the initial $\mu_i$ and $Z_i$ were used (i.e., not averages) the flow rate would be 369 Mscf/d, a deviation of 14%.

<table>
<thead>
<tr>
<th>$p_e$ (psi)</th>
<th>$\mu$, cp</th>
<th>$Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3,000</td>
<td>0.0199</td>
<td>0.9115</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p_{wf}$ (psi)</th>
<th>$\mu$, cp</th>
<th>$\bar{\mu}$, cp</th>
<th>$Z$</th>
<th>$\bar{Z}$</th>
<th>$q$, Mscf/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.0146</td>
<td>0.0173</td>
<td>0.963</td>
<td>0.937</td>
<td>356</td>
</tr>
<tr>
<td>1,000</td>
<td>0.0153</td>
<td>0.0176</td>
<td>0.934</td>
<td>0.923</td>
<td>324</td>
</tr>
<tr>
<td>1,500</td>
<td>0.0162</td>
<td>0.0181</td>
<td>0.913</td>
<td>0.912</td>
<td>270</td>
</tr>
<tr>
<td>2,000</td>
<td>0.0173</td>
<td>0.0186</td>
<td>0.902</td>
<td>0.907</td>
<td>195</td>
</tr>
<tr>
<td>2,500</td>
<td>0.0186</td>
<td>0.0193</td>
<td>0.9019</td>
<td>0.907</td>
<td>104</td>
</tr>
</tbody>
</table>
3.3.2 Transient Flow

At early time the flowing bottomhole pressure of a producing well is a function of time if the rate is held largely constant. This type of flow condition is called transient flow and is used deliberately during a pressure transient test. In practice, the well is usually operated under the same wellhead pressure (which is imposed by the well hardware such as chokes, etc.), the resulting flowing bottomhole pressure is also largely constant, and the flow rate will vary with time. To characterize gas flow in a reservoir under transient conditions, the combination of the generalized Darcy’s law (rate equation), and the continuity equation can be used (in radial coordinates)

\[
\phi \frac{\partial \rho}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( \rho \frac{k \partial \rho}{\mu} \right),
\]

where \( \phi \) is the porosity. Because gas density is a strong function of pressure (in contrast to oil, which is considered incompressible), the real gas law can be employed, and as shown in Eq. (1.9) in Chapter 1.

Therefore,

\[
\phi \frac{\partial}{\partial t} \left( \frac{p}{Z} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k p}{\mu Z} \frac{\partial p}{\partial r} \right).
\]

In an isotropic reservoir with constant permeability, Eq. (3.15) can be simplified to

\[
\phi \frac{\partial}{k \partial t} \left( \frac{p}{Z} \right) = \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{p}{\mu Z} \frac{\partial p}{\partial r} \right).
\]

Performing the differentiation on the right-hand side of Eq. (3.16), assuming that the viscosity and gas deviation factor are small functions of pressure, and rearranging, it gives

\[
\frac{\phi \mu}{k p} \frac{\partial^2 p}{\partial t} = \frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r} \frac{\partial p^2}{\partial r}.
\]

For an ideal gas, \( c_g = 1/p \), and as a result, Eq. (3.17) leads to

\[
\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r} \frac{\partial p^2}{\partial r} = \frac{\phi \mu c}{k} \frac{\partial p^2}{\partial t}.
\]
This approximation looks exactly like the classic diffusivity equation for oil. Its solution would look exactly like the solutions of the equation for oil, but instead of \( p \), the pressure squared, \( p^2 \), should be used as a reasonable approximation.

Al-Hussainy and Ramey (1966) used a far more appropriate and exact solution by employing the real gas pseudopressure function, defined as

\[
m(p) = 2\int_{p_o}^{p} \frac{p}{\mu Z} dp,
\]

where \( p_o \) is some arbitrary reference pressure (usually zero). The differential pseudopressure, \( \Delta m(p) \), defined as \( m(p) - m(p_{wf}) \), is then the driving force in the reservoir.

Using Eq. (3.19) and the chain rule

\[
\frac{\partial m(p)}{\partial t} = \frac{\partial m(p)}{\partial p} \frac{\partial p}{\partial t} = \frac{2p}{\mu Z} \frac{\partial p}{\partial t}.
\]

Similarly,

\[
\frac{\partial m(p)}{\partial r} = \frac{2p}{\mu Z} \frac{\partial r}{\partial r}.
\]

Therefore, Eq. (3.16) becomes

\[
\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r} \frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_i}{k} \frac{\partial m(p)}{\partial t}.
\]

The solution of Eq. (3.22) would look exactly like the solution to the diffusivity equation cast in terms of pressure. Dimensionless time is (in oilfield units):

\[
t_D = \frac{0.000264 kt}{\phi (\mu c_i) r_w^2},
\]

and dimensionless pressure is

\[
p_D = \frac{kh[m(p_i) - m(p_{wf})]}{1,424 qT}.
\]
Equations (3.22) to (3.24) suggest solutions to natural gas problems (e.g., well testing) that are exactly analogous to those for an oil well, except now it is the real gas pseudopressure function that needs to be employed. This function is essentially a physical property of natural gas, dependent on viscosity and the gas deviation function. Thus, it can be readily calculated for any pressure and temperature by using standard physical property correlations.

By analogy with oil, transient rate solution under radial infinite acting conditions can be written as:

\[
q = \frac{kh[m(p_i) - m(p_{wf})]}{1,638T} \left[ \log t + \log \frac{k}{\phi(\mu c_i) r_w^2} - 3.23 + 0.87s \right]^{-1}, \tag{3.25}
\]

where \( q \) is gas flow rate in Mscf/d and \( c_i \) is the total compressibility of the system. As usual Eq. (3.25) can be cast in terms of pressure squared difference

\[
q = \frac{k [p_i^2 - p_{wf}^2]}{1,638 \mu Z T} \left[ \log t + \log \frac{k}{\phi(\mu c_i) r_w^2} - 3.23 + 0.87s \right]^{-1}. \tag{3.25a}
\]

Equations (3.25) or (3.25a) can be used to generate transient IPR (Inflow Performance Relationship) curves for a gas well. Transient behavior ends when boundaries are felt. A commonly accepted expression for the time in hours when pseudosteady state begins is

\[
t_{pss} \approx 1,200 \frac{\phi \mu c_i r_w^2}{k}. \tag{3.26}
\]

**Example 3–2 Rate at the onset of pseudosteady state**

Use the well in Example 3–1 and calculate the production rate at the time when pseudosteady begins and also at one tenth the time. Use a flowing bottomhole pressure of 1,500 psi. The gas saturation in the reservoir is about 0.75 and the porosity is 0.25.

**Solution**

First, estimate the time to pseudosteady state using the expression given above. The gas compressibility at initial conditions can be cal-
culated from Eq. (1.17) but at a relatively low pressure of 3,000 psi it can be approximated by

\[ c_s = \frac{1}{3,000} \approx 3.33 \times 10^{-4} \text{ psi}^{-1}. \]

Therefore the total compressibility is approximately equal to

\[ c_t = S_g c_s = 0.75 \times 3.33 \times 10^{-4} = 2.5 \times 10^{-4} \text{ psi}^{-1}. \]

The time to pseudosteady state, using Eq. (3.26) and the data of Example 3–1 and Table 3–2 is then

\[ t_{pss} = 1,200 \times \frac{0.25 \times 0.0199 \times 2.5 \times 10^{-4} \times 660^2}{0.1} = 6,500 \text{ hr}. \]

Then using Eq. (3.25a) for 6,500 hours

\[ q = \frac{0.1 \times 50 \times [3,000^2 - 1,500^2]}{1,638 \times 0.0181 \times 0.913 \times 710} \]

\[ \left[ \log 6,500 + \log \frac{0.1}{0.25 \times 0.0199 \times 2.5 \times 10^{-4} \times 0.359^2 - 3.23} \right]^{-1} \]

\[ = 276 \text{ Mscf/d}. \]

After 650 hours the rate would be 328 Mscf/d.

### 3.4 Gas Well Inflow under non-Darcy Flow

All expressions given thus far in this chapter have ignored one of the most important effects in natural gas flow: turbulence. For very low permeability reservoirs in mature environments such as the United States and continental Europe, it is sufficient to assume that gas flow in the reservoir obeys Darcy’s law as we did in the previous section. Newly found reservoirs are primarily offshore, in developing nations, and are of moderate to high permeability, i.e., 1 to 100 md.

As well deliverability increases, turbulence becomes increasingly dominant in the production of gas wells. For reservoirs whose permeability is more than 5 md, turbulence effects may account for a 20 to
60% reduction in the production rate of an openhole well (when laminar flow is assumed). Turbulence in such cases practically overwhelms all other factors, including damage (Wang and Economides, 2004). In this section, turbulence effects in a vertical well will be discussed.

### 3.4.1 Turbulent Flow in Gas Wells

As mentioned earlier in this chapter, turbulent flow has been studied since the 1900s (Forchheimer, 1914). Pioneering and prominent among a number of investigators in the petroleum literature have been Katz and co-workers (Katz et al., 1959; Firoozabadi and Katz, 1979; Tek et al., 1962). They suggested that turbulence plays a considerable role in well performance, showing that the production rate is affected by itself; the larger the potential rate, the larger the relative detrimental impact would be. Since most turbulent flow takes place near the wellbore region, the effect of turbulence provides an extra pressure drop as given by

\[ p_e^2 - p_{wf}^2 = \frac{1.424 \mu Z T}{kh}[\ln\left(\frac{r_e}{r_w}\right) + s]q + \frac{1.424 \mu Z T D}{kh}q^2, \quad (3.27) \]

where \( D \) is the turbulence coefficient with units of reciprocal rate. Eq. (3.27) can be rearranged and turbulence can be accounted for by a rate-dependent skin effect as described by (Swift and Kiel, 1962)

\[ q = \frac{kh(p_e^2 - p_{wf}^2)}{1.424 \mu Z T[\ln(\frac{r_e}{r_w}) + s + Dq]}, \quad (3.28) \]

Similarly, the same turbulence coefficient can be employed to the more rigorous expressions using the real-gas pseudopressure. As an example, for pseudosteady state with \( q \) in Mscf/d

\[ q = \frac{kh(p_e^2 - p_{wf}^2)}{1.424 \mu Z T[\ln(0.472 \frac{r_e}{r_w}) + s + Dq]}, \quad (3.28a) \]

or

\[ q = \frac{kh[m(\bar{p}) - m(p_{wf})]}{1.424 T[\ln(0.472 \frac{r_e}{r_w}) + s + Dq]} \quad (3.28b) \]
3.4 Gas Well Inflow under non-Darcy Flow

$D$ is usually determined by analysis of multi-rate pressure tests (Economides et al., 1994; Kakar et al., 2004), or from correlations when well test data is not available. In the absence of field measurements, an empirical relation is proposed (Economides et al., 1994)

$$D = \frac{6 \times 10^{-5} \gamma k_s^{-0.1} h}{\mu r_w h_{\text{perf}}} ,$$  

(3.29)

where $h_{\text{perf}}$ is the perforated section length in ft and $k_s$ is the near-wellbore permeability in md.

**Example 3–3 Gas well rate with non-Darcy effects**

A gas well produces from a reservoir whose pressure is 3,150 psi, and the reservoir temperature is 148°F. Gas specific gravity is 0.61 with no sour gases. The net pay is 50 ft. The damage skin factor is equal to 5 and the reservoir permeability is 20 md. The non-Darcy coefficient $D$ is 1.5E-3 (Mscf/d)$^{-1}$. Calculate the rate of the well at $p_{\text{wfp}} = 1,200$ psi assuming pseudosteady state. Also assume that: $\ln \left(0.472 r_e/r_w\right) = 7$. What is the apparent skin at that rate? What would be the miscalculated rate if the non-Darcy effects were ignored?

**Solution**

Use Lee et al. (1966) and Dranchuk et al. (1974) correlations (described in Chapter 1) to calculate viscosity, $Z$-factor, and $m(p)$. The calculated PVT data is summarized in Table 3–3.

Using Eq. (3.28b), the gas well production rate would be

$$m(3,150) - m(1,200) = \frac{1,424 \times 608}{20 \times 50} (7 + 5)q + \frac{1,424 \times 608 \times 0.0015}{20 \times 50} q^2 .$$

Substituting the values of the real-gas pseudopressure from Table 3–3 and simplifying, the following quadratic equation is obtained

$$q^2 + 8,000q - 4.41 \times 10^8 = 0 .$$

The solution is 17,380 Mscf/d. The apparent skin equals

$$s + Dq = 5 + (1.5E-3) \times 17,380 = 31.$$
Chapter 3  Natural Gas Production

For a skin equal to 5 the rate would be more than 55,000 Mscf/d, if non-Darcy effects are ignored (i.e. $D = 0$).

### 3.4.2 Correlations for Turbulence in Vertical Gas Well

Figure 3–3 is a sketch of a vertical gas well and its cross section. It is obvious that when the flow is far away from the wellbore, the flow velocity is small, and the flow can be assumed as laminar. In the near wellbore area, fluid converges to the small diameter production tubing. Turbulence occurs especially when the permeability is high and the well deliverability increases.

In radial gas flow wells, well performance can be described by (Katz et al., 1959)

\[
p_e^2 - p_{wf}^2 = \frac{1.424 \mu Z T}{kh} \left( \ln \left( \frac{r_e}{r_w} \right) + s \right) q + \frac{3.16 \times 10^{-12} \beta_\gamma' S T (\frac{1}{r_w} - \frac{1}{r_e})}{h^2} \left( \frac{1}{r_w} + \frac{1}{r_e} \right) q^2 ,
\]

Table 3–3  PVT Table for Example 3–3

<table>
<thead>
<tr>
<th>$p$ (psia)</th>
<th>$Z$</th>
<th>$\mu$ (cp)</th>
<th>$p/(\mu Z)$ Interval</th>
<th>$\Delta p$</th>
<th>$p/(\mu Z) \times \Delta p$</th>
<th>$2 \times (p/(\mu Z) \times \Delta p)$</th>
<th>$m(p)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.998</td>
<td>0.0127</td>
<td>1,159.80</td>
<td>14.7</td>
<td>8.52E+03</td>
<td>1.70E+04</td>
<td>1.70E+04</td>
</tr>
<tr>
<td>14.7</td>
<td>0.960</td>
<td>0.0130</td>
<td>32,051.28</td>
<td>1.66E+04</td>
<td>6.40E+06</td>
<td>1.28E+07</td>
<td>1.28E+07</td>
</tr>
<tr>
<td>400</td>
<td>0.925</td>
<td>0.0135</td>
<td>64,064.06</td>
<td>4.81E+04</td>
<td>1.92E+07</td>
<td>3.84E+07</td>
<td>5.13E+07</td>
</tr>
<tr>
<td>8,000</td>
<td>0.895</td>
<td>0.0143</td>
<td>93,760.99</td>
<td>7.89E+04</td>
<td>3.16E+07</td>
<td>6.31E+07</td>
<td>1.14E+08</td>
</tr>
<tr>
<td>1,200</td>
<td>0.873</td>
<td>0.0152</td>
<td>120,576.40</td>
<td>1.07E+05</td>
<td>4.29E+07</td>
<td>8.57E+07</td>
<td>2.00E+08</td>
</tr>
<tr>
<td>1,600</td>
<td>0.873</td>
<td>0.0152</td>
<td>143,554.40</td>
<td>1.32E+05</td>
<td>5.28E+07</td>
<td>1.06E+08</td>
<td>3.06E+08</td>
</tr>
<tr>
<td>2,000</td>
<td>0.860</td>
<td>0.0162</td>
<td>155,532.80</td>
<td>1.50E+05</td>
<td>3.74E+07</td>
<td>7.48E+07</td>
<td>3.81E+08</td>
</tr>
<tr>
<td>2,250</td>
<td>0.856</td>
<td>0.0169</td>
<td>164,810.90</td>
<td>1.60E+05</td>
<td>4.00E+07</td>
<td>8.01E+07</td>
<td>4.61E+08</td>
</tr>
<tr>
<td>2,500</td>
<td>0.857</td>
<td>0.0177</td>
<td>172,847.30</td>
<td>1.69E+05</td>
<td>4.22E+07</td>
<td>8.44E+07</td>
<td>5.45E+08</td>
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<tr>
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<td>0.0185</td>
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<td>1.76E+05</td>
<td>4.40E+07</td>
<td>8.80E+07</td>
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<td>1.81E+05</td>
<td>2.72E+07</td>
<td>5.44E+07</td>
<td>6.87E+08</td>
</tr>
</tbody>
</table>
3.5 Horizontal Gas Well Inflow

Horizontal Gas Well Inflow

[Diagram of an openhole vertical well and its cross section]

Figure 3–3  A sketch of an openhole vertical well and its cross section

where $k$ equals the horizontal permeability, $k_H$, $\beta$ is the Katz et al. version of non-Darcy coefficient, and can be calculated by using the Tek et al. (1962) correlation listed in Table 3–1.

The discussion above is for openhole vertical well radial flow. Turbulent flow in perforated cased wells has been addressed elsewhere (Wang and Economides, 2004; Karakas and Tariq, 1988; Ichara, 1987).

In summary, for higher-permeability natural gas reservoirs, turbulence may become the dominant influence on production. For vertical wells, the accounting for turbulence is relatively well understood and inflow equations have been adjusted to account for the phenomenon. Furthermore, field-testing techniques have been established to obtain the non-Darcy coefficient. Surprisingly, similar work has not yet been done for horizontal wells. This will be detailed in the following section.

3.5 Horizontal Gas Well Inflow

Horizontal wells outside of the former Soviet Union started in the 1980s, and eventually, were widely introduced in the early 1990s. Since then, they have proliferated and have become essential in oil and gas production (Economides and Martin, 2007). The main advantages of horizontal wells are (Joshi, 1991; Cho and Shah, 2001):

- To increase productivity as the wellbore is longer than that of vertical well.
- To reduce water or gas coning.
- To reduce turbulence in gas wells (emphasis ours).
- To intersect fractures in naturally fractured reservoirs and drain reservoirs more effectively.
- To improve drainage area per well and reduce the number of vertical wells in low permeability reservoirs.
- To increase injectivity of an injection well and enhance sweep efficiency.

There are quite a few important publications related to horizontal well performance (Celier et al., 1989; Dikken, 1990; Joshi, 1991; Norris et al., 1991; Ozkan et al., 1999; Economides et al., 1994; Cho and Shah, 2001), but few have addressed turbulence effects on well performance. Of those that discussed turbulence, most assumed that turbulence is small and can be neglected. Their assumption is that the horizontal well length \( L \) is much longer compared to the vertical well height \( h \), and therefore, they concluded that turbulence is smaller in horizontal wells compared to vertical wells and could be ignored. This is true when the reservoir is isotropic and the permeability is small. But when permeability increases, well deliverability increases, and turbulence effects can no longer be neglected. Based on a recent study, the production loss due to turbulence could account for 30% in horizontal wells. When the reservoir is anisotropic, it is much worse (Wang and Economides, 2009).

Joshi (1991) whose contributions in the understanding of horizontal well performance have been seminal also attempted to quantify turbulence effects in natural gas horizontal wells. He developed (for a pseudosteady state) a horizontal well equation using a vertical well analog

\[
q = \frac{k_{h} h (p^2 - p_{wf}^2)}{1,424 \mu Z T (\ln (r_c / r_w) - 0.75 + s + s_m + s_{CA} + Dq - c')},
\]

where \( s \) is the horizontal well equivalent skin effect that would be imposed on a vertical well, \( s_m \) is mechanical (damage) skin, \( s_{CA} \) is shape related skin, and \( c' \) is a shape constant.

Eq. (3.31) is correct for oil but not for gas where turbulence is important. In fact, it is quite wrong. It uses horizontal well equivalent skins that can only be correct under reservoir flow, such as a pseudo-radial into a vertical well. Then the turbulence effects are presumed to influence flow far away from the well. Indeed the equivalent horizontal well skin under turbulent gas conditions cannot be
the same as for oil wells. By assuming so, and with such skins invari-
ably of large negative values, it is no wonder that the effects of tur-
bulence have been underestimated by Joshi and others who have used
his solution.

Diyashev and Economides (2006) calculated vertical well equiva-
lent skins for horizontal wells by using an expression derived from
Joshi’s own horizontal well equation

\[
s = -\ln \left[ \frac{L}{4r_w} \left( \frac{1}{I_{ani} h/r_w (I_{ani} + 1)} \right)^{I_{ani} h/L} \right].
\]  (3.32)

Using Eq. (3.32), negative values of the skin can be as much as \(-8\)
for long horizontal wells in favorite anisotropy settings. Introducing
such number in the denominator of Eq. (3.31) would certainly under-
estimate the impact of turbulence. In reality, the expression inside the
bracket in Eq. (3.32) should have the \(Dq\) term added, which would
change the equivalent skin by 30 to 50%.

Wang and Economides (2009) conducted a study to investigate
properly the turbulence effects in horizontal wells. They presented
appropriate correlations to account for turbulence effects on horizontal
well performance, and offered a large range of parametric studies that
involve reservoir thickness, permeability anisotropy, porosity, and hor-
izontal well length. Their approach follows.

Analogs to Eq. (3.11) (for steady state), the inflow performance
relationships (IPR) for a nonfractured horizontal well in a gas reser-
voir follows (Joshi, 1991; Economides et al., 1994).

For steady state:

\[
q = \frac{k_h \left( p_e^2 - p_w^2 \right)}{1,424 \mu Z T \left( \ln \left( \frac{a + \sqrt{a^2 - (L/2)^2}}{L/2} \right) + \frac{I_{ani} h}{L} \left( \ln \frac{I_{ani} h}{r_w (I_{ani} + 1)} + Dq \right) \right)}.
\]  (3.33)

For pseudosteady state:

\[
q = \frac{k_h \left( \bar{p}^2 - p_w^2 \right)}{1,424 \mu Z T \left( \ln \left( \frac{a + \sqrt{a^2 - (L/2)^2}}{L/2} \right) + \frac{I_{ani} h}{L} \left( \ln \frac{I_{ani} h}{r_w (I_{ani} + 1)} - \frac{3}{4} + Dq \right) \right)}.
\]  (3.34)
Or, replacing the approximation \( (\bar{p}^2 - p_{wf}^2) / \mu Z \) by the real-gas pseudopressure difference

\[
q = \frac{k_H h(m(\bar{p}) - m(p_{wf}))}{1,424 T \left( \ln \left\{ \frac{a + \sqrt{a^2 - (L/2)^2}}{L/2} \right\} + \frac{I_{ani} h}{L} \left\{ \ln \frac{I_{ani} h}{r_w(I_{ani} + 1) - \frac{3}{4} + Dq} \right\} \right)}, \tag{3.35}
\]

where \( k_H \) is the horizontal permeability and \( L \) is the horizontal well length. \( I_{ani} \) is a measurement of vertical-to-horizontal permeability anisotropy and is given by

\[
I_{ani} = \sqrt{\frac{k_H}{k_V}}, \tag{3.36}
\]

where \( k_H \) is defined as \( \sqrt{k_x k_y} \) and \( k_V \) equals to \( k_z \). \( a \) is the large half-axis of the drainage ellipsoid formed by a horizontal well length, \( L \). The expression for this ellipsoid is

\[
a = \frac{L}{2} \left\{ 0.5 + \left[ 0.25 + \left( \frac{r_{eh}}{L/2} \right)^{4.5} \right]^{0.5} \right\}^{0.5} \quad \text{for} \quad \frac{L}{2} < 0.9 r_{eh}, \tag{3.37}
\]

where \( r_{eh} \) is the drainage radius in the horizontal wells.

The correlation of the non-Darcy coefficient, developed by Tek et al. (1962) and listed in Table 3–1, is valid for natural gas flow through porous media. Therefore, it can be used in a horizontal well by making the following adjustment

\[
k = \sqrt[3]{k_x k_y k_z} = \sqrt[3]{k_H k_V} \cdot \tag{3.38}
\]

So the turbulence factor in a horizontal well is

\[
\beta_H = \frac{5.5 \times 10^9}{(k_x k_y k_z)^{5/12} \phi^{3/4}} \cdot \tag{3.39}
\]

The turbulence coefficient for a horizontal well is
where \( r_{wH} \) is the effective wellbore radius of the horizontal wells and is equal to

\[
r_{wH} = \frac{r_w (1 + I_{ani})}{2I_{ani}}. \tag{3.41}
\]

With the correlations developed above, the well inflow for horizontal wells with turbulence can be examined.

**Example 3–4**  Gas horizontal well performance with turbulence

Calculate turbulence effects in the horizontal well and compare the results with those from the vertical well. The input parameters are given in Table 3–4. Assume skin is zero. Reservoir permeability is 0.1, 1, 10, and 100 md, respectively.

| \( p_e \)   | 3,000 psi |
| \( p_{wf} \) | 1,500 psi |
| \( r_e \)   | 2,978 ft  |
| \( r_w \)   | 0.359 ft  |
| \( h \)     | 50 ft     |
| \( L \)     | 1,000 ft  |
| \( T \)     | 710 R     |
| \( \phi \)  | 18%       |
| \( \mu \)   | 0.0162 cp |
| \( Z \)     | 0.91      |
| \( \gamma_s \) | 0.7      |
Solution

With the procedure outlined above, the flow rates from both horizontal and vertical gas wells with (actual) and without (ideal) turbulence can be calculated. Results are summarized in Table 3–5.

Results show that the production in the ideal openhole horizontal well is about 3.4 times higher than that in the vertical well (assuming no turbulence effects). At the same drawdown, it is obvious that the productivity in the horizontal well is higher than that in the vertical well, as the horizontal well has a longer wellbore.

When turbulence is taken into account, production in both horizontal and vertical wells drops especially when the permeability is high. When permeability is less than 1 md, the impact of turbulence in the horizontal well is less than 2% while it is less than 5% in the vertical well. When permeability increases there is a greater reduction in the production rate. When the permeability is 100 md, as shown in Figure 3–4, the production loss due to turbulence effect climbs to 30% and 40% for the horizontal and vertical wells, respectively. Even with turbulence effect, the horizontal well still performs better than the ideal vertical well. At 100 md permeability, the production from the actual horizontal well (with turbulence) is 2.4 times higher than that from the ideal openhole vertical well (without turbulence).

When comparing the performance between the actual horizontal and vertical wells, the results are even more promising. The horizontal well production is 3.4 times the vertical well at 1 md and this climbs to 3.9 at 100 md, which is higher than the ideal productivity ratio between the horizontal and vertical wells (3.3 at 1 md and 3.4 at 100 md). This shows that, at the given parameters, the horizontal well is the desirable option over the vertical well in terms of reducing turbulence and increasing production, but the effects of turbulence are clearly not negligible.

This effect is even more profound when the formation is anisotropic. Assume the horizontal permeability is 10 md, the vertical permeability is 10, 1, and 0.1 respectively. These values give the index of permeability anisotropy, \( I_{ani} = \sqrt{\frac{k_H}{k_V}} \) as 10, 3, and 1, respectively. All other parameters are the same as those given in Table 3–4. Repeating the same calculation as done in Example 3–4, results are summarized in Table 3–6. The actual rates are not that interesting but the ratios are more profound, and are plotted in Figure 3–5.

It is obvious that horizontal well deliverability is very sensitive to the reservoir anisotropy when compared with the performance of the
This is because the controlling permeability in the horizontal well is a function of the horizontal and vertical permeabilities as shown in Eq. (3.33), while the vertical well performance depends only on the horizontal permeability. Thus, when the horizontal permeability is kept constant (here it is 10 md), the vertical well production is constant (shown in Table 3–6), and the reduction due to turbulence is about 13% (Figure 3–5).

### Table 3–5 Results for Example 3–4

<table>
<thead>
<tr>
<th>$k_r$ (md)</th>
<th>Vertical Ideal $q_{Ideal ; OH}$, MMscf/d ($\beta = 0$, $s = 0$)</th>
<th>Vertical Actual $q_{Radial ; Flow}$, MMscf/d ($\beta &gt; 0$, $s = 0$)</th>
<th>Horizontal Ideal $q_{Ideal ; OH}$, MMscf/d ($\beta = 0$, $s = 0$)</th>
<th>Horizontal Actual $q_{Radial ; Flow}$, MMscf/d ($\beta &gt; 0$, $s = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.3</td>
<td>0.3</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>1</td>
<td>2.5</td>
<td>2.4</td>
<td>8.4</td>
<td>8.3</td>
</tr>
<tr>
<td>10</td>
<td>25.1</td>
<td>21.9</td>
<td>84.2</td>
<td>77.5</td>
</tr>
<tr>
<td>100</td>
<td>250.9</td>
<td>158.0</td>
<td>841.2</td>
<td>609.6</td>
</tr>
</tbody>
</table>

### Figure 3–4 Turbulence effects in both horizontal and vertical wells
The production reduction in the horizontal well due to turbulence, on the other hand, changes significantly when the reservoir becomes more anisotropic (from 0.9 to 0.7 shown in Figure 3–5). The production ratio between horizontal and vertical wells is 3.4, 2.8, and 1.8 for the ideal case, and 3.1, 2.2, and 1.2 for the actual horizontal over ideal vertical case at $I_{ani}$ of 1, 3, and 10, respectively. When comparing the production between the actual horizontal and vertical wells, it shows the ratio changes from 3.5 to 2.5 and 1.4 when $I_{ani}$ varies from 1 to 3 and 10, respectively. Important conclusions can be

**Figure 3–5 Effects of index of permeability anisotropy**

The production reduction in the horizontal well due to turbulence, on the other hand, changes significantly when the reservoir becomes more anisotropic (from 0.9 to 0.7 shown in Figure 3–5). The production ratio between horizontal and vertical wells is 3.4, 2.8, and 1.8 for the ideal case, and 3.1, 2.2, and 1.2 for the actual horizontal over ideal vertical case at $I_{ani}$ of 1, 3, and 10, respectively. When comparing the production between the actual horizontal and vertical wells, it shows the ratio changes from 3.5 to 2.5 and 1.4 when $I_{ani}$ varies from 1 to 3 and 10, respectively. Important conclusions can be

<table>
<thead>
<tr>
<th>$I_{ani}$</th>
<th>Vertical Ideal $q_{Ideal \ OH}$, MMscf/d ($\beta = 0, s = 0$)</th>
<th>Vertical Actual $q_{Radial \ Flow}$, MMscf/d ($\beta &gt; 0, s = 0$)</th>
<th>Horizontal Ideal $q_{Ideal \ OH}$, MMscf/d ($\beta = 0, s = 0$)</th>
<th>Horizontal Actual $q_{Radial \ Flow}$, MMscf/d ($\beta &gt; 0, s = 0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25.1</td>
<td>21.9</td>
<td>84.2</td>
<td>77.5</td>
</tr>
<tr>
<td>3</td>
<td>25.1</td>
<td>21.9</td>
<td>70.4</td>
<td>54.4</td>
</tr>
<tr>
<td>10</td>
<td>25.1</td>
<td>21.9</td>
<td>46.2</td>
<td>30.8</td>
</tr>
</tbody>
</table>

Table 3–6 Effects of Index of Permeability Anisotropy
drawn by comparing the results. For isotropic formations, horizontal wells alleviate turbulence more effectively than vertical wells, showing a larger productivity index ratio than the ideal cases (3.5 versus 3.4). However, as anisotropy increases (e.g., $I_{ani} = 10$) horizontal wells become less efficient to reduce turbulence effects (real versus ideal productivity ratios of 1.4 versus 1.8). In this particular case, turbulence can reduce production in horizontal wells by 30% when permeability is anisotropic.

Turbulence effect in the horizontal well is also a function of reservoir thickness, porosity, and horizontal well length. Detailed discussion can be found in Wang and Economides (2009).

In summary, turbulence effects are the dominant features in the production of high (>5 md) permeability gas wells. Turbulence may account for a 25 to 50% reduction in the expected openhole production rate from such vertical gas wells (Wang and Economides, 2004). In a horizontal well, turbulence effect cannot be neglected as many people have proposed in the past. On the contrary, turbulence effects dominate horizontal well flow in higher permeability reservoirs. In fact, in permeability anisotropic formations they reduce the flow rate by a larger fraction than in vertical wells. Porosity, which was part of the original turbulence correlations, mysteriously disappears from more recently published correlations. It is reintroduced in the correlations in this chapter, as its impact is considerable especially when the permeability is anisotropic (Wang and Economides, 2009).

There are several ways to reduce turbulence in high rate gas wells. One way is to perforate wellbores with long penetrating perforation tunnels and large perforation densities (e.g., 8 to 12 SPF). However, nothing can compete with hydraulic fracturing. In higher permeability gas wells, the incremental benefits greatly exceed those of comparable permeability oil wells. This is because of the dramatic impact on reducing the turbulence effects beyond the mere imposition of a negative skin. It is fair to say that any gas well above 5 md will be greatly handicapped if not hydraulically fractured. In fact, pushing the limits of hydraulic fracturing by using large quantities of premium proppants will lead to extraordinary production rate increases.

### 3.6 Hydraulic Fracturing

A widely used technique for production enhancement is hydraulic fracturing, which involves the creation of a crack in the reservoir by injecting highly pressurized fluids at a very high rate. The fluids are solutions of polymers, which are used to thicken the carrier fluid, often water, for the purpose of increasing its viscosity and allowing it
to carry particles, called proppants. The hydraulically created fracture is held open (propped) with tens of thousands to millions of pounds of clean, uniform natural sand or synthetic materials, and can have a permeability that is orders of magnitude larger than the surrounding reservoir, creating something equivalent to a super highway.

### 3.6.1 Hydraulic Fracturing Overview

Hydraulic fracturing started in the late 1940s and has evolved into the second largest investment (after drilling) of the oil and gas industry. From right before 2000 to 2008, the fracturing industry grew from $2.8 billion to $12.8 billion, representing an average increase of ±21% per year. No other petroleum activity showed such increase (Energy Tribune, 2008).

During the first 40 years, hydraulic fracturing was applied almost exclusively to low permeability reservoirs. However, starting in the late 1980s and increasingly in the 1990s, it encompassed any permeability reservoirs, including ones of extremely high permeability such as 200 to as high as 2,000 md. The important development was the ability to perform a tip screenout (TSO). Since unrestricted fracturing would generate both unwanted length and cause inordinate leakoff, a TSO arrests the fracture growth and inflates the fracture to the desired width. As seen below, far shorter but wider fractures are indicated for higher permeability reservoirs and such geometry can be accomplished only through a TSO.

In many writings, we have defined low and high permeability reservoirs for hydraulic fracturing as those where the design of the treatment execution would require TSO or not, respectively. For oil reservoirs below 5 md, the execution can be as an unrestricted fracture, hence they are low permeability. For 50 md and higher a TSO is necessary. For intermediate permeability, a TSO may not be necessary but often is used.

For natural gas wells, these permeability values are an order of magnitude smaller. Low permeability reservoirs are below 0.5 md and those above 5 md should be considered as high permeability formations (Economides et al. 2002a). (Note to the reader: Since the authors have been involved with a recent book specifically dealing with hydraulic fracturing of natural gas wells, the text below will be only an anthology of important concepts, emphasizing production related issues. A far more in-depth analysis can be found in Economides and Martin, 2007.)
Before delving into hydraulic fracturing, it is necessary to review the concept of dimensionless productivity index, as it will be used extensively later in this chapter.

### 3.6.2 The Concept of Dimensionless Productivity Index

The dimensionless productivity index, $J_D$, warrants some definition. The relationship between the dimensioned productivity index (PI) and the dimensionless $J_D$ of an oil well is simply

$$
\frac{q}{p - p_{wf}} = \frac{kh}{\alpha_i B \mu} J_D,
$$

where the constant $\alpha_i$ is the familiar 141.2 in the traditional oilfield units or 18.4 if $q$ (m$^3$/d), $p$ (atm) and $h$ (m).

For natural gas wells the analogous expression is

$$
\frac{q}{p^2 - p_{wf}^2} = \frac{kh}{\alpha_i Z T} J_D,
$$

where the constant $\alpha_i$ is the familiar 1,424 for oilfield units.

In Eqs. (3.42 and 3.43), the reservoir pressure, $p$, is either the constant outer boundary pressure, $p_e$, for steady state, or the average (and declining) reservoir pressure, $\bar{p}$, for pseudosteady state. The $J_D$ is well known by familiar expressions for steady state radial flow in a vertical well

$$
J_D = \frac{1}{\ln \left( \frac{r_e}{r_w} \right) + s},
$$

or, for pseudosteady-state flow

$$
J_D = \frac{1}{\ln \left( \frac{r_e}{r_w} \right) - 0.75 + s}.
$$

For a nondamaged well, the $J_D$ would range between 0.11 and 0.13 for almost all drainage and wellbore radii combinations in both steady state and pseudosteady state. Thus, $J_D$ values around 0.1 denote undamaged wells. Smaller values denote damage; larger values denote stimulation such as hydraulic fracturing, or more favorable geometry such as horizontal or complex well architecture (Diyashev and Economides, 2006).
3.6.3 Unified Fracture Design (UFD)

Valkó, Economides, and coworkers such as Romero et al. (2002), introduced a physical optimization technique to maximize the productivity index of a hydraulically fractured well that they have called the Unified Fracture Design (UFD) approach.

Central to the UFD is the Proppant Number, $N_{\text{prop}}$, given by

$$N_{\text{prop}} = I_x^2 C_{fD} = \frac{4k_f x_f w}{k x_w^2} = \frac{4k_f x_f w h_p}{k x_w^2 h_p} = \frac{2k_f V_p}{k V_r},$$ (3.46)

where $I_x$ is the penetration ratio and $C_{fD}$ is the dimensionless fracture conductivity, $V_r$ is the reservoir drainage volume, and $V_p$ is the volume of the proppant in the pay. It is equal to the total volume injected times the ratio of the net height to the fracture height. $k_f$ is the proppant pack permeability and $k$ is the reservoir permeability.

For gas wells, the nominal proppant pack permeability is reduced to an effective permeability because of turbulence effects in the fracture. How this adjustment is done will be shown in a later section.

The idea of UFD is that fracturing transcends permeability, and for a given value of $N_{\text{prop}}$ there exists a unique geometry involving the fracture length and width (and therefore an optimum fracture conductivity) that would maximize well performance. Any other fracture conductivity, and therefore any other design, would lead to a lower well performance.

As shown by Economides et al. (2002a), at Proppant Numbers less than 0.1 the optimal conductivity, $C_{fD} = 1.6$. At larger Proppant Numbers, the optimum conductivity increases and the absolute maximum for the dimensionless productivity index, $J_D$ is $6/\pi = 1.909$.

While graphical representations of these concepts can be found in the previously mentioned references, Valkó and Economides (1996) also presented correlations for the maximum achievable dimensionless productivity index as a function of the Proppant Number

$$J_{D_{\text{max}}} (N_{\text{prop}}) = \begin{cases} 
1 & \text{if } N_{\text{prop}} \leq 0.1 \\
0.990 - 0.5 \ln N_{\text{prop}} & \\
\frac{6}{\pi} - \exp \left[ \frac{0.423 - 0.311 N_{\text{prop}} - 0.089 (N_{\text{prop}})^2}{1 + 0.667 N_{\text{prop}} + 0.015 (N_{\text{prop}})^2} \right] & \text{if } N_{\text{prop}} > 0.1
\end{cases}$$ (3.47)
The optimal dimensionless fracture conductivity for the entire range of Proppant Numbers is given by

\[
C_{D,\text{opt}}(N_{\text{prop}}) = \begin{cases} 
1.6 & \text{if } N_{\text{prop}} < 0.1 \\
1.6 + \exp\left[\frac{-0.583 + 1.48\ln N_{\text{prop}}}{1 + 0.142\ln N_{\text{prop}}}\right] & \text{if } 0.1 \leq N_{\text{prop}} \leq 10 \\
N_{\text{prop}} & \text{if } N_{\text{prop}} > 10
\end{cases}
\] (3.48)

With the optimal dimensionless fracture conductivity determined, then the optimal fracture length and width are set, and they represent the only ones for which the fracture must be designed

\[
x_{\text{opt}} = \left(\frac{k_f V_f}{C_{D,\text{opt}} k h}\right)^{0.5} \quad \text{and} \quad w_{\text{opt}} = \left(\frac{C_{D,\text{opt}} k V_f}{k_f h}\right)^{0.5},
\] (3.49)

where \(V_f\) is the volume of one propped wing, \(V_f = V_p/2\).

UFD is an essential means to optimize fractured well performance and post-treatment evaluation can be made against design expectations. More to the point is that improvements in design, increasing proppant volumes, and using higher quality materials can be accomplished through the employment of these techniques. They can “push the limits” of hydraulic fracturing to levels unthinkable earlier (Demarchos et al., 2004).

Using a set of constraints such as a limit of 1,000 psi net pressure during execution (affecting directly the resulting fracture width), a minimum hydraulic fracture width of at least 3 times the proppant diameter to prevent proppant bridging, and an injection time of no more than 24 hours; Economides et al. (2004) developed a benchmarking graph for the maximum attainable \(J_D\) for oil wells for a range of permeabilities, shown in Figure 3–6. This representation is significant because it suggests what extraordinary results can be achieved by pushing the limits of design and using large volumes of higher quality proppant, while still respecting operational and logistical constraints.

One of the most striking conclusions of UFD and pushing the limits of fracturing is: If better proppants are used with higher \(k_f\), the indicated propped width of the fracture is smaller, allowing longer
fractures for a given mass of proppant. Thus, much larger treatments can be executed before a net pressure constraint is in effect. This is counter to conventional practices, where better proppants have been sold to perform smaller treatments, and achieve similar results as those using lower quality proppants such as natural sand, resulting in the saving of a miniscule amount of money, while foregoing huge increases in production.

Example 3–5 Optimized fractured well performance

Use the following well, reservoir, and fracture treatment data. Calculate maximum $J_D$, optimum $C_{pD}$, and indicated fracture geometry (length and width). Apply to two different permeabilities: 1 and 100 md. In this example ignore the effects of turbulence. What would be the folds of increase between fractured and nonfractured wells?

- Drainage area (square) = $4.0E + 6$ ft$^2$ (equivalent drainage radius for radial flow = 1,130 ft)
- Mass of proppant = 200,000 lb
- Proppant specific gravity = 2.65
- Porosity of proppant = 0.38
- Proppant permeability = 220,000 md (20/40 ceramic)
Net thickness = 50 ft
Fracture height = 100 ft

**Solution**

First, the volume of the proppant in the pay is \[200,000 \times \frac{50/100}{(2.65 \times 62.4 \times (1 – 0.38))}\] = 975 ft.

Then for \(k = 1 \text{ md}\) from Eq. (3.46)

\[N_{prop} = \frac{2 \times 220,000 \times 975}{1 \times 2 \times 10^8} = 2.1.\]

Using the lower part of Eq. (3.47), \(J_D\) maximum is then 1.1. From Eq. (3.48) \(C_{fD,opt} = 2.5\).

Therefore from Eq. (3.49)

\[x_{fopt} = \left(\frac{220,000 \times 975 / 2}{2.5 \times 1 \times 50}\right)^{0.5} = 920 \text{ ft},\]

and

\[w_{opt} = \left(\frac{2.5 \times 1 \times 975 / 2}{220,000 \times 50}\right)^{0.5} = 0.0105 \text{ ft} = 0.13 \text{ in.}\]

For \(k = 100 \text{ md}\) from Eq. (3.46), the Proppant Number is 100 times smaller (0.021), and as should be expected, \(C_{fD,opt} = 1.6\). (No need to calculate). From Eq. (3.47), maximum \(J_D\) is then 0.34. From Eq. (3.49) \(x_{fopt}\) and \(w_{opt}\) are 115 ft and 1 in., respectively.

Given that the \(J_D\) of a nonfractured well would be 0.135 (from Eq. (3.44) and using \(r_w = 0.328 \text{ ft}\)). The folds of increase for the two wells would be 8.2 and 2.5, respectively.

### 3.6.4 Performance of a Hydraulically Fractured Well with Turbulence

Economides et al. (2002b) presented an iterative procedure combining the UFD method with the Gidley (1990) adjustment to the
proppant pack permeability, and the Cooke (1993) correlations for flow in fractures, to account for the enhanced turbulence effects in fracture flow. It must be emphasized that while turbulence in the fracture reduces the would-be performance, the overall improvement in well production is very large when compared to that of a nonfractured well because of the enhanced turbulence effects in high permeability radial flow (Marongiu-Porcu et al., 2008).

The nominal proppant pack permeability is corrected to an effective value using the Reynolds number in the fracture by

\[ k_{f,e} = \frac{k_{f,n}}{1 + N_{Re}} \]  

(3.50)

where \( k_{f,n} \) is the nominal fracture permeability.

There is an indicated iterative procedure and it starts by assuming a Reynolds number. An obvious first value for the Reynolds number is zero, which means that the nominal proppant pack permeability is not affected by turbulence and is equal to the effective permeability. Then, after adjusting with Eq. (3.50), the Proppant Number is calculated from Eq. (3.46), and the maximum \( J_D \) (Eq. (3.47) and the optimum dimensionless conductivity (Eq. (3.48) are calculated. The latter allows the determination of the indicated fracture dimensions using Eq. (3.49).

For the rest of this calculation, there are additional needed variables compared to designing fractures for oil wells or for low permeability gas wells. The determined dimensionless productivity index and the well drawdown allow the determination of the expected production rate, which in turn is used to calculate the velocity in the fracture and to obtain the Reynolds number. The procedure ends when the assumed and calculated Reynolds numbers are close enough.

The Reynolds number for non-Darcy flow is given by

\[ N_{Re} = \frac{\beta k_{f,n} \nu \rho}{\mu} \]  

(3.51)

where \( k_{f,n} \) is the nominal permeability (under Darcy flow conditions) in \( m^2 \), \( \beta \) is in \( 1/m \), \( \nu \) is the fluid velocity at reservoir conditions in the fracture in \( m/s \), \( \mu \) is the viscosity in \( Pa.s \), and \( \rho \) is the density in \( kg/m^3 \). The value of \( \beta \) is obtained from
\[ \beta = (1 \times 10^8) \frac{b}{(k_{f,nc})^a}, \] (3.52)

where \( a \) and \( b \) are obtained from Cooke (1993). The values of \( a \) and \( b \) for common proppant sizes are given in Table 3–7.

### Table 3–7  Constants \( a \) and \( b \)

<table>
<thead>
<tr>
<th>Prop Size</th>
<th>( a )</th>
<th>( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 to 12</td>
<td>1.24</td>
<td>17,423</td>
</tr>
<tr>
<td>10 to 20</td>
<td>1.34</td>
<td>27,539</td>
</tr>
<tr>
<td>20 to 40</td>
<td>1.54</td>
<td>110,470</td>
</tr>
<tr>
<td>40 to 60</td>
<td>1.6</td>
<td>69,405</td>
</tr>
</tbody>
</table>

**Example 3–6** Optimized fractured well performance with turbulence

Repeat Example 3–5 for the 100 md case, but now consider the effects of turbulence in both the nonfractured and fractured wells. Calculate the folds of increase under pseudosteady-state conditions.

Additional variables are:

\[ \bar{p} = 3,000 \text{ psi} \]
\[ p_{wf} = 1,500 \text{ psi} \]
\[ T = 250^\circ F = 710 \text{ R} \]
\[ \gamma = 0.7 \]

and thus at 1,500 psi, \( Z = 0.91 \), and \( \mu = 0.0162 \text{ cp} \),

and at 3,000 psi, \( Z = 0.91 \), and \( \mu = 0.02 \text{ cp} \)

\[ D = 3.3 \times 10^{-5} \text{ (Mscf/d)}^{-1} \] for radial flow.

**Solution**

Applying the pseudosteady version of Eq. (3.28) and substituting variables

\[ q^2 + 2.23 \times 10^5 q = 6.15 \times 10^{10} \]
and thus, \( q = 160,000 \text{ Mcf/d}. \) Ignoring turbulence effects this flow rate would be 276,000 Mcf/d.

For the fractured well and without correcting for turbulence effects, using \( J_D = 0.34 \) from Example 3–5 (i.e., \( N_{Re} = 0 \)),

\[
q = \frac{kh(\bar{p}^2 - p_w^2)}{1,424\mu ZT} J_D = \frac{100 \times 50 \times (3,000^2 - 1,500^2)}{1,424 \times 0.018 \times 0.91 \times 710} \times 0.34 = 693,000 \text{ Mcf/d}.
\]

This rate is 2.5 times the rate for radial flow uncorrected for turbulence (276,000 Mcf/d) as found in Example 3–5. However, turbulence cannot be ignored and the procedure outlined in the earlier section must be followed.

The formation volume factor can be obtained from Eq. (1.12) and is calculated at the flowing bottomhole condition

\[
B_g = 0.0283 \times \frac{0.91 \times 710}{1,500} = 0.012(\text{res ft}^3 / \text{scf}).
\]

The density can be calculated using Eq. (1.10)

\[
\rho_g = 2.7 \times \frac{1,500 \times 0.7}{0.91 \times 710} = 4.83 \text{ lb/ft}^3 = 77.4 \text{ kg/m}^3.
\]

And finally, the velocity can be determined by (using 1 in. width as calculated in Example 3–5 and dividing by 2 for the two wings of the fracture):

\[
\nu = (0.012 \times 693,000 \times 1,000) / [24 \times 3,600 \times 100 \times (1/12) \times 2] \]
\[
= 5.8 \text{ ft/sec} = 1.77 \text{ m/s}.
\]

From Eq. (3.52) and using \( a = 1.54 \) and \( b = 110,470 \) for 20/40 mesh proppant (from Cooke correlation, Table 3–7)

\[
\beta = (1 \times 10^8) \times \frac{110,470}{(220,000)^{1.54}} = 6.54 \times 10^4 / \text m.
\]

And finally, from Eq. (3.51)

\[
N_{Re} = \frac{6.54 \times 10^4 \times 2.17 \times 10^{-10} \times 1.77 \times 77.4}{0.0162 \times 10^{-3}} = 120.
\]
Clearly, the assumed (zero) and calculated Reynolds numbers are quite different.

An instructive second iteration would be for \( N_{Re} = 9 \), which would reduce the effective permeability by a factor of ten as per Eq. (3.50), in this Example to 22,000 md. The Proppant Number becomes ten times smaller than the one calculated in Example 3–5 (0.0021), and again, \( C_{fD,opt} = 1.6 \). From Eq. (3.47), \( J_D \) maximum is then 0.25. From Eq. (3.49), \( x_{fopt} \) and \( w_{opt} \) are 36 ft and 3.2 in., respectively. (Note in practice such large width may be unrealistic but is used here for illustration purposes.)

With the new \( J_D \), the rate is 510,000 Mscf/d and the new velocity is now 0.41 m/s. From Eq. (3.51), \( N_{Re} = 27.8 \). It is still different from the assumed value of nine.

Convergence occurs at \( N_{Re} = 18 \) with maximum \( J_D = 0.23 \), new rate = 470,000 Mscf/d. The effective proppant pack permeability is 11,600 md, and \( x_{fopt} \) and \( w_{opt} \) are 26 ft and 4.5 in., respectively.

Some very important lessons are learned from this Example. The reduction in effective permeability results in a demand for a much larger width (and in this case, one that may not be able to be achieved in the field, but very aggressive designs may approach these widths). More important, is that the ratio of the productivity indexes between the fractured and the nonfractured wells, when considering turbulence effects, is now 470,000/160,000 = 3 (versus. 2.5); showing the considerable impact of fracturing in remedying turbulence.

Marongiu-Porcu et al. (2008) presented an important study comparing the folds of productivity index increase between fractured and nonfractured wells for both oil and gas. Figure 3–7 is the comparison, and the results show the major impact of turbulence in gas wells. First, for oil wells, the folds of increase are predictable. As the reservoir permeability increases, the folds of PI increase are reduced. For example, while at 0.1 md, the folds of increase are over 10, and at 100 md they are only 2. For gas wells at small reservoir permeabilities, the trends are similar to oil, but as the reservoir permeability increases, the folds of PI increase take an upward trend. This is because of the enhanced turbulence effects in radial flow and the considerable reduction of turbulence in the fractured wells. Figure 3–7 is one of the most important indicators that while for oil wells one may make the case that fracturing in high permeability wells may not be compelling (i.e. in some cases horizontal wells may be better than fractured vertical wells); however, for gas wells hydraulic fracturing is absolutely essential in any range of permeabilities. (Note: In Figure 3–7 the fracture width is as wide as determined from the optimum values of \( J_D \) and \( C_{fD} \).)
3.6.5 Fracturing Horizontal Gas Wells

In anticipation of hydraulic fracturing, horizontal wells can be drilled either along the maximum or the minimum horizontal stress orientations, thus, executed fractures will be longitudinal or transverse, respectively. The performance of a longitudinally fractured horizontal well is almost identical to a fractured vertical well when both have equal fracture length and equal conductivity. Therefore, existing solutions for vertical well fractures can be applied to a longitudinally fractured horizontal well (Valkó and Economides, 1996; Soliman et al., 1999; Economides and Martin, 2007).

The interesting new element is the ability to perform multiple transverse fracturing treatments with proper zonal isolation and spacing. The vast majority of applications of fractured horizontal wells are for transverse fractures. The configuration of a transversely fractured horizontal well is demonstrated in Figure 3–8, and it provides a visualization of the process and challenges. The cross section of the contact between a transverse fracture and a horizontal well is $2\pi r_w w$ where $w$ is the width of the fracture (which can be obtained by using a design procedure such as the Unified Fracture Design approach) and $r_w$ is the radius of the horizontal well. Figure 3–8 shows the flow from the reservoir into the fracture is linear while the flow inside the fracture is converging radial. This combination of flows results in an additional pressure drop which can be accounted for by a skin effect, denoted as $s_c$ (Mukherjee and Economides, 1991).
Therefore, the design procedure for each transverse fracture employs the UFD, which allows for the calculation of $J_{D,max}$ and $s_c$. This in turn leads to the dimensionless productivity index of each transverse fracture (neglecting for now turbulence effects), $J_{DTH}$:

$$J_{DTH} = \frac{1}{\frac{1}{J_{DV}} + s_c},$$

where $J_{DV}$ is the $J_{D,max}$ of the fractured vertical well.

With $J_{DTH}$ and drawdown, the actual production rate can be obtained using

$$q = \frac{kh(p^2 - p_{wf}^2)}{1,424\mu ZT} J_{DTH}. \quad (3.55)$$

For gas wells, the iterative procedure outlined in the previous subsection for the performance of fractured vertical wells also applies to transversely fractured horizontal wells. The obvious difference is that turbulence effects will be more pronounced because of the far reduced contact between well and fracture and the cross-sectional
area of flow. For a vertical well the flow area would be $2wh_p$, whereas for a transversely fractured horizontal well, it would be $2\pi r_w w$. For the same width the cross-sectional area of flow of a vertical well would be 100 to 200 times larger ($h_f / \pi r_w$).

Turbulence effects have a great impact on transversely fractured horizontal gas wells due to the small cross section of the contact between the well and the fracture. Because of the impact of turbulence effects, the results for the permeability range of 1 md to 100 md, which performs very well in vertical fractured gas wells, are unacceptable in transversely fractured horizontal gas wells. Marongiu-Porcu et al. (2009) have demonstrated that only a very small range of reservoir permeabilities in gas wells lends itself to the transverse fracture configuration, i.e., $0.1 < k < 0.5$. The conclusion is based on both physical and economic considerations. For larger permeability values, turbulence effects reduce fracture performance (even with multiple fractures such as ten treatments) to unacceptable production rates and vertical wells become preferable. For the lower permeability range, outside of North America, where treatment costs are significantly lower than the rest of the world, the expected production rates are not sufficient to warrant the drilling of horizontal wells and their subsequent well completion and fracturing.

**Example 3–7 Performance of transversely fractured horizontal well**

Calculate the flow rate in a transversely fractured horizontal well (with one transverse fracture) for formation permeability of 0.1, 1, 10, and 100 md. Relevant well data are given as below:

- Nominal proppant permeability = 600,000 md
- Mass of proppant = 400,000 lbm
- Porosity of proppant pack = 0.3
- Specific gravity of proppant = 3.27
- Net thickness = 50 ft
- Well radius = 0.359 ft
- Well drainage radius = 660 ft
- Pretreatment skin factor = 0
- Fracture height = 100 ft
- Gas specific gravity (air = 1) = 0.7
3.6 Hydraulic Fracturing

3.6.1 The calculation procedure is outlined in Figure 3–9. In calculating the Reynolds number with Eq. (3.51) in this Example, the velocity is determined by dividing the downhole volumetric flow rate by the cross-sectional area of flow as explained in the subsection above. This greatly increases turbulence effects in a transverse fracture at any permeability but particularly at higher permeability (see results in Figure 3–10).

For comparison purposes, the flow rate from the ideal openhole vertical well (without turbulence), radial vertical well (actual with turbulence), and vertical fractured well are also calculated. The productivity ratio (against the ideal openhole vertical well) is plotted in Figure 3–10.

Results show that when permeability is 0.1, turbulence is negligible. The fold of increase (FoI) from a single transversely fractured
horizontal well is about 3.4. FoI from a fractured vertical well is ~13. That is almost four times higher than in the transversely fractured horizontal well, which means that four or more treatments in a horizontal well would result in higher performance than a vertical well/vertical fracture configuration.

Once the permeability is higher than 1 md, the choke and turbulence effects in the transversely fractured horizontal well become dominating. The skin, $s_c$ (described in Eq. (3.53), increases from 0.6 at 0.1 md to 6.7 at 1 md and 137 at 100 md (shown in Figure 3–11). This causes the FoI from the single transversely fractured horizontal well to be less than 1, which means its performance is worse than that in an ideal vertical openhole well ($\beta = 0, s = 0$). When permeability is 100 md the FoI drops to 0.05. The FoI from the vertical fractured well is over 2. It would take 40 transverse treatments ($2/0.05$) in a horizontal well to equal the performance of one vertical well/vertical fracture.

This example suggests that transversely fractured horizontal wells, even with a large number of treatments (and ignoring the economic cost), simply cannot compete physically with vertical fractured wells when the permeability is higher than, e.g., 0.5 md (even when premium proppant such as 600,000 md) is used.
3.7 Well Deliverability

“Deliverability” of a gas well is defined as a production rate into the wellbore, and subsequently, along the production tubing to the surface facilities. In underground storage or enhanced recovery, deliverability also relates to the rate at which gas can be injected from a well into the reservoir (Lee et al., 1984). The flow rate from a drainage area into a wellbore is a function of the properties of both the formation and the fluids, as well as the prevailing gradients of driving forces (Lee et al., 1987).

To perform well deliverability calculations, the pressure drop in a gas well must be determined. The unique aspect is that the fluid is compressible and the fluid density and fluid velocity vary along the pipe. These variations must be included when integrating the mechanical energy balance equation which, with no shaft work and neglecting kinetic energy changes, is

$$\frac{dp}{\rho} + \frac{g}{g_c}dz + \frac{2f_c u^2 dL}{g_c D} = 0,$$

(3.56)

where $f_c$ is the Fanning friction factor. It can be obtained from the Moody friction chart (Moody, 1944) or the Chen equation (Chen, 1979).
where \( \varepsilon \) is the relative pipe roughness. \( N_{Re} \) is the Reynolds number and its calculation is discussed later in this section.

Since \( dz \) in Eq. (3.56) is \( \sin \theta dL \) (see demonstration in Figure 3–12), the last two terms can be combined as

\[
\frac{1}{\sqrt{f_f}} = -4 \log \left\{ \frac{\varepsilon}{3.7065} - \frac{5.0452}{N_{Re}} \log \left[ \frac{\varepsilon^{1.1098}}{2.8257} + \left( \frac{7.149}{N_{Re}} \right)^{0.8981} \right] \right\}, \quad (3.57)
\]

Replacing \( \rho \) by Eq. (1.10), the fluid velocity can be determined using the real gas law and be related to the well flow rate given in standard conditions, \( q \),

\[
u = \frac{4}{\pi D^2} q Z \left( \frac{T}{T_{sc}} \right) \left( \frac{p_{sc}}{p} \right).
\] (3.59)

Thus, Eq. (3.58) yields

\[
\frac{ZRT}{28.97 \gamma_s p} dp + \left\{ \frac{g}{g_c} \sin \theta + \frac{32f_f}{\pi^2 g_c D^3} \left[ \left( \frac{T}{T_{sc}} \right) \left( \frac{p_{sc}}{p} \right) qZ \right]^2 \right\} dL = 0.
\] (3.60)
3.7 Well Deliverability

Eq. (3.60) requires numerical integration to be solved properly. However, if an average temperature is used in an interval and if, also, an average value of the gas deviation factor, \( Z \), for the interval is used then Eq. (3.60) can be integrated for nonhorizontal flow to yield

\[
p_2^2 = e^s p_1^2 + \frac{32 f_l}{\pi^2 D^5 g_c \sin \theta} \left( \frac{Z T q p_{sc}}{T_{sc}} \right)^2 (e^s - 1),
\]

where \( s \) is defined as

\[
s = \frac{-2 \times 28.97 \gamma_s (g / g_c) \sin \theta L}{ZRT}.
\]

For horizontal flow, \( \sin \theta \) and \( s \) are zero; integration of Eq. (3.60) gives

\[
p_1^2 - p_2^2 = \frac{64 \times 28.97 \gamma_s f_l ZT}{\pi^2 g_c D^5 R} \left( \frac{p_{sc} q}{T_{sc}} \right)^2 L.
\]

For each interval, an estimate of the average \( \bar{Z} \) can be obtained as a function of the average temperature, \( \bar{T} \), and the known pressure, \( p_1 \). After the pressure, \( p_2 \), is calculated, the assumed \( \bar{Z} \) can be compared with the calculated value using \( \bar{T} \) and the average pressure, \((p_1 + p_2)/2\). Iteration may be necessary in some cases.

To complete the calculation, the friction factor must be obtained from the Reynolds number and the pipe roughness. Since the product, \( \rho \mu \), is a constant for flow of a compressible fluid, \( N_{Re} \) can be calculated based on standard conditions as

\[
N_{Re} = \frac{4 \times 28.97 \gamma_s q p_{sc}}{\pi D \rho \mu R T_{sc}^2}.
\]

The viscosity should be evaluated at the average temperature and pressure as was the compressibility factor, \( \bar{Z} \).

Eq. (3.60) for vertical flow and in oilfield units becomes

\[
p_2^2 = e^s p_1^2 + 2.685 \times 10^{-3} \frac{f_l (ZT q)^2}{\sin \theta D^5} (e^s - 1),
\]
Chapter 3 Natural Gas Production

or

\[ p_1^2 = e^{-s} p_2^2 - 2.685 \times 10^{-3} \frac{f_s (ZTq)^2}{\sin \theta D^5} (1 - e^{-s}), \]  

(3.66)

if the flowing bottomhole pressure \((p_1)\) is the unknown and will be calculated from the surface pressure of \(p_2\). In Eqs. (3.65 and 3.66), \(s\) is defined as

\[ s = -0.0375 \gamma_s \frac{\sin \theta L}{ZT}. \]  

(3.67)

Eq. (3.62) for horizontal flow becomes

\[ p_1^2 - p_2^2 = 1.007 \times 10^{-4} \frac{\gamma_s f_s ZT q^2 L}{D^5}. \]  

(3.68)

Finally the Reynolds number becomes

\[ N_{Re} = 20.09 \frac{\gamma_s q}{D \mu}. \]  

(3.69)

In Eqs. (3.65 to 3.69), \(p\) is in psia, \(q\) is in Mscf/d, \(D\) is in inches, \(L\) is in ft, \(\mu\) is in cp, and \(T\) is in R.

Example 3–8 Wellbore hydraulics and pressure calculations

A well flows 10 MMscf/d of natural gas from a depth of 13,000 ft with a 3-in. tubing in a vertical well. At the surface, the temperature is 150°F and the pressure is 650 psia; the bottomhole temperature is 230°F. The gas gravity is 0.7 and the relative roughness of the tubing is 0.0006. Calculate the flowing bottomhole pressure at the given rate. Repeat the calculation for 20 MMscf/d and show what tubing diameter would be required to produce the same flowing bottomhole pressure.

What would the rate be for a 3-in. pipe if the wellhead pressure is 650 psia and the flowing bottomhole pressure cannot exceed 2,000 psi?
Solution

Eqs. (3.66, 3.67, and 3.69) are needed to solve this problem.

Using the average temperature, 650 R, and using the known pressure at the surface as the average pressure (for now), 650 psia, with the given gas gravity, and the assumption of zero percent of sour gases; the average $Z$-factor and gas viscosity can be obtained from the correlations in Chapter 1 as $\bar{Z} = 0.936$ and $\bar{\mu} = 0.0137 \text{ cp}$.

From Eq. (3.69), the Reynolds number is,

$$N_\text{Re} = \frac{20.09 \times 0.7 \times 10,000}{3 \times 0.0137} = 3.42 \times 10^6,$$

and with roughness of 0.0006, using the Chen equation (Eq. (3.57)) leads to $f_f = 0.0044$. Since the flow direction is vertical upward, $\theta = +90^\circ$.

Now using Eq. (3.67),

$$s = \frac{-0.0375 \times 0.7 \times \sin(90^\circ) \times 130,000}{0.936 \times 650} = -0.56.$$

The bottomhole pressure is calculated from Eq. (3.66)

$$p_1^2 = e^{-(-0.602)} \times 650^2 - 2.685 \times 10^{-3} \times \frac{0.0044 \times (0.875 \times 650 \times 10,000)^2}{\sin(90^\circ) \times 3^5} (1 - e^{-(-0.602)})$$

and thus, $p_1 = p_{\text{wf}} = 1,445 \text{ psia}$.

Readjusting the average pressure to $(1,445 + 640)/2 = 1,048 \text{ psi}$, new $\bar{Z}$ and $\bar{\mu}$ are obtained and the above calculation is repeated. The final results are $\bar{Z} = 0.90$, $\bar{\mu} = 0.014$, $N_\text{Re} = 3.25 \times 10^6$, $f_f = 0.044$, $s = -0.58$, and the flowing bottomhole pressure at 10 MMscf/d is $p_1 = p_{\text{wf}} = 1,440 \text{ psia}$.

Doubling the rate to 20 MMscf/d would require a flowing bottomhole pressure equal to 2,431 psi.

For a flow rate of 20 MMscf/d, a wellhead pressure of 650 psi, and a bottomhole pressure of 1,440 psi, the required tubing diameter would be 4 in.

For the 3-in. pipe with two pressure constraints (650 and 2,000), the flow rate is 15.8 MMscf/d.
Example 3–9  Gas well deliverability

A natural gas well produces from a depth of 13,000 ft with a 3-in. tubing in a vertical well. The surface temperature is 150°F and the pressure is 650 psia; the bottomhole temperature is 230°F. The gas gravity is 0.7 and the relative roughness of the tubing is 0.0006 (this information is the same as for Example 3–8).

If the reservoir permeability is 1 md, the pay thickness is 75 ft, and the reservoir pressure is 6,000 psi:

1. Determine the well deliverability.
2. Repeat the calculation for a ten-fold larger permeability of 10 md.
3. Determine what tubing diameter would be required to produce the same flowing bottomhole pressure in the second reservoir as for the first.

Solution

Using the same procedure outlined in Example 3–8, for the first question the flowing rate is about 12 MMscf/d at the corresponding flowing bottomhole pressure of 1,650 psi. By using the same procedure, the tubing performance curve is generated for a range of potential rates.

The IPR curve was obtained from the Swift and Kiel (1962) pseudosteady-state model Eq. (3.28), while the non-Darcy coefficient $D$ has been estimated to be approximately equal to $10^{-4}$ (Mscf/d)$^{-1}$ by using the correlation given by Eq. (3.29). Graphical solution of this case is presented in Figure 3–13.

For a permeability of 10 md and all other input data unchanged, a flowing rate of about 38.5 MMscf/d is obtained at the corresponding flowing bottomhole pressure of 4,530 psi. Graphical solution of this case is presented in Figure 3–14.

The results of Figure 3–14 are significant. First, it is clear that the production rate is not even close to a ten-fold increase over the 1 md reservoir case. The reasons are the much large turbulence effects in the reservoir, and as important, the pressure drops in the tubing. Note the almost 3-fold increase in the required flowing bottomhole pressure. Clearly this well is tubing limited.

For the same inflow condition determined in Question 2, the tubing diameter required to produce the same flowing bottomhole pressure of Question 1 (1,650 psi) is 6.3 in., which also produces a new flowing rate of about 79 MMscf/d. These results show the importance of proper tubular designs in high rate natural gas wells. (Note:
3.8 Forecast of Well Performance and Material Balance

Forecast of well performance is intended to predict well deliverability, adding the very important variable of time. Production under steady state is simple. Assuming that a well can be maintained at roughly

![Graphical solution of this case is presented in Figure 3–15.](image1)

**Figure 3–13** Well deliverability for Example 3–9, \( k = 1 \text{ md}, D_{tbg} = 3 \text{ in.} \)

![Graphical solution of this case is presented in Figure 3–15.](image2)

**Figure 3–14** Well deliverability for Example 3–9, \( k = 10 \text{ md}, D_{tbg} = 3 \text{ in.} \)

the calculated tubing diameter is theoretical. In practice, a standard tubing size would be used, e.g., 6 in.) Graphical solution of this case is presented in Figure 3–15.

3.8 Forecast of Well Performance and Material Balance

Forecast of well performance is intended to predict well deliverability, adding the very important variable of time. Production under steady state is simple. Assuming that a well can be maintained at roughly
steady state because of e.g., strong bottom water drive, then the pro-
duction rate will remain largely constant for as long as the condition
is maintained. Under transient conditions, forecast of well perfor-
enance is also relatively easy. The intersection of transient IPR’s with
the well vertical lift performance curve will provide the expected pro-
duction rates versus time. Transient well performance will be in force
if the reservoir permeability is quite low and, thus boundary effects
will take time to appear.

Of unique interest is the forecast of well performance under pseu-
dosteady state conditions for which material balance is necessary.

If \( G_i \) and \( G \) are the initial and current gas-in-place in standard
conditions within a drainage area, the difference between the two of
them is the cumulative production from a gas reservoir, as a result of
fluid expansion and, thus

\[
G_p = G_i - G = G_i - G_i \frac{B_{g_i}}{B_g}
\]  

(3.70)

where \( B_{g_i} \) and \( B_g \) are the initial and current formation volume factors,
respectively.

Eq. (1.12) in Chapter 1 provides \( B_g \) in terms of pressure, tempera-
ture, and the gas deviation factor. Substitution in Eq. (3.70) for iso-
thermal conditions, which is a reasonable assumption, and
rearrangement results in

\[
G_p = G_i \left( 1 - \frac{\bar{p}}{p_i} / \frac{Z}{Z_i} \right).
\]  

(3.71)
Eq. (3.71) is one of the best known expressions in reservoir and production engineering, and it suggests that a plot of $G_p$, the cumulative production, in the abscissa, $\overline{p}/Z$ and in the ordinate, should form a straight line. At $G_p=0$, $\overline{p}/Z = p_i/Z_i$, and at $\overline{p}/Z = 0$, $G_p = G_i$. For any value of the reservoir pressure (and associated $Z$), there exists a corresponding $G_p$.

The indicated well performance forecast procedure follows.

First, a reservoir pressure decline increment is assumed, e.g., 500 psi. The resulting average pressure (and the easy to calculate $\overline{p}/Z$) would lead to the cumulative recovery for the interval. Next, the production rate for the interval can be determined, using the pseudosteady state relationships presented earlier in this chapter (Eq. (3.14) without turbulence effects and Eq. (3.29) with turbulence effects), employing the average reservoir pressure of the interval and the well deliverability methods outlined in the last section. The time for each interval would then be simply $\Delta G_p/q$.

---

Example 3–10 Forecast of gas well performance under pseudosteady state

Present a forecast of production, reservoir pressure, and cumulative recovery as a function of time. The same natural gas well that was used in Examples 3–8 and 3–9 (depth 13,000 ft, with 3-in. tubing ID, surface temperature 150°F, surface pressure 650 psia, reservoir temperature 230°F, gas gravity 0.7) drains 160 acres with porosity equal to 0.2, and water saturation equal to 0.3. The reservoir permeability is 1 md, the pay thickness is 75 ft, and the initial reservoir pressure is 6,000 psi.

Abandonment reservoir pressure is 2,000 psi.

Solution

The first step is to calculate the initial $Z$-factor, which is equal to 1.08, and therefore $p_i/Z_i = 5,560$ psi.

Then, the initial gas-in-place is calculated

$$G_i = 160 \times 43,560 \times 75 \times 0.2 \times (1 - 0.3) / 3.5 \times 10^{-3} = 20.9 \times 10^9 \text{ scf} = 20.9 \text{ Bcf},$$

where the initial formation volume factor, $B_{gi} = 3.5 \times 10^{-3}$ res ft$^3$/scf.

Figure 3–16 is the graphical depiction of the material balance whose algebraic expression in Bcf is $G_p = 20.9 - 0.00375 \overline{p}/Z$.

One round of calculations is shown next.
Chapter 3  Natural Gas Production

Assume the reservoir pressure declines to 5,500 psi. Then $Z = 1.04$ and $\bar{p}/Z = 5,290$ psi. The cumulative recovery, $G_p$, is then (from Figure 3–16) 1.06 Bcf.

Then, using a deliverability calculation as shown in Example 3–9, ignoring turbulence, and with an average reservoir pressure of $(6,000 + 5,500)/2 = 5,750$ psi, the flow rate $q = 13.5$ MMcf/d. Therefore $G_p/q = 79$ days.

Table 3–8 contains all the calculations for this exercise. The production rate, reservoir pressure, and cumulative production versus time are plotted in Figure 3–17.

The material balance, depicted in Figure 3–16, can be constructed before production starts. It can be based on the initial pressure build up test, from which the initial reservoir pressure will be determined, and on geological information of drainage area, reservoir net thickness, porosity, and water saturation.

During production, if the original assumption was correct, then a plot of actual cumulative production versus $p/Z$ (also determined from successive pressure build up tests) should fall exactly on the original material balance curve. Otherwise, if the points are to the left of the initial curve, they would extrapolate to a lower $G_p$, suggesting smaller drainage area or smaller reservoir net thickness.

Conversely, if the actual data are to the right of the initial curve, this would invariably suggest strong bottom water drive, in which case the entire construction is not really valid.

Figure 3–16  Material balance for Example 3–10

Assume the reservoir pressure declines to 5,500 psi. Then $Z = 1.04$ and $\bar{p}/Z = 5,290$ psi. The cumulative recovery, $G_p$, is then (from Figure 3–16) 1.06 Bcf.

Then, using a deliverability calculation as shown in Example 3–9, ignoring turbulence, and with an average reservoir pressure of $(6,000 + 5,500)/2 = 5,750$ psi, the flow rate $q = 13.5$ MMcf/d. Therefore $G_p/q = 79$ days.

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Conversely, if the actual data are to the right of the initial curve, this would invariably suggest strong bottom water drive, in which case the entire construction is not really valid.
Figure 3–17  Production rate, reservoir pressure, and cumulative recovery for Example 3–10


Table 3–8 Material Balance Calculations for Example 3–10

<table>
<thead>
<tr>
<th>$p$, psi</th>
<th>$Z$</th>
<th>$p/Z$, psi</th>
<th>$G_p$, Bcf</th>
<th>$\Delta G_p$, Bcf</th>
<th>$q_p$, MMcf/d</th>
<th>$\Delta t$, days</th>
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3.9 References


